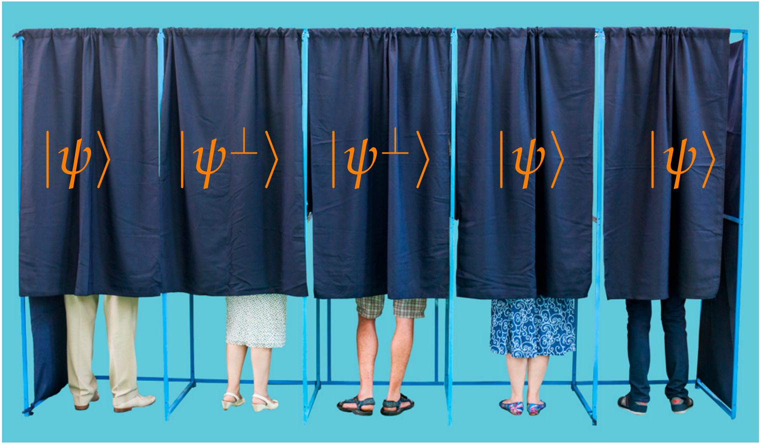
Quantum majority vote



Māris Ozols

QuSoft & University of Amsterdam

Contents

Quantum majority vote

Joint work with:

- Harry Buhrman *(QuSoft & CWI)*
- Laura Mančinska (*University of Copenhagen*)
- Noah Linden (*University of Bristol*)
- Ashley Montanaro (*Phasecraft & University of Bristol*)

Linear programming with unitary-equivariant constraints

Joint work with:

- Dmitry Grinko (*QuSoft & University of Amsterdam*)

Manifesto

- All information is quantum…

Manifesto

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- …there is no classical information

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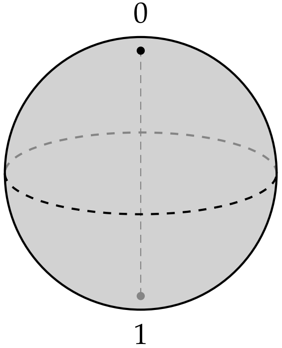

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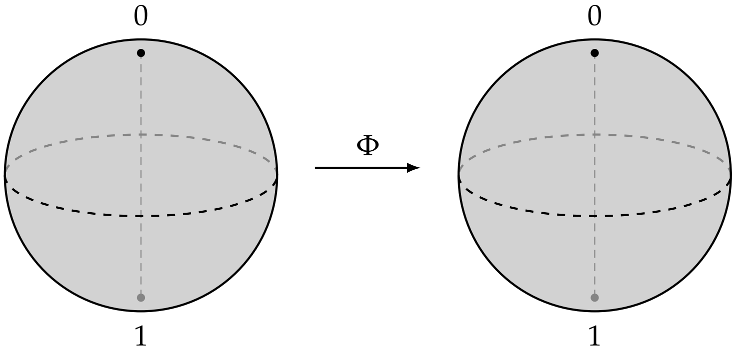
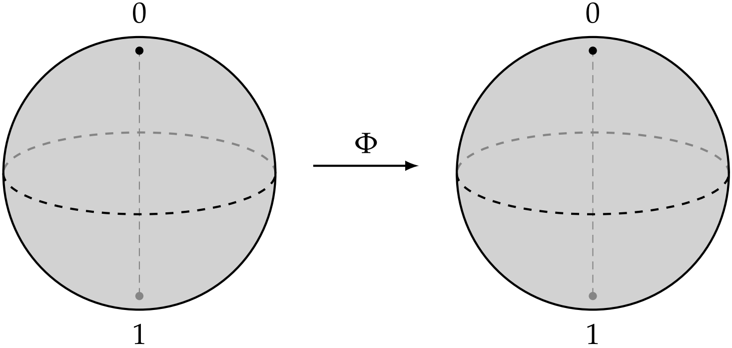


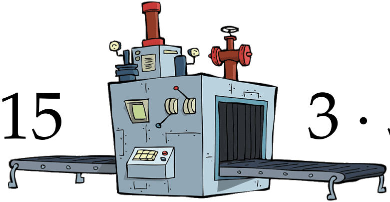
Manifesto

- All information is quantum…
- …there is no classical information
- Algorithm = CPTP map
- Classical computer science is dead!



Manifesto (lite)

15



$3 \cdot 5$

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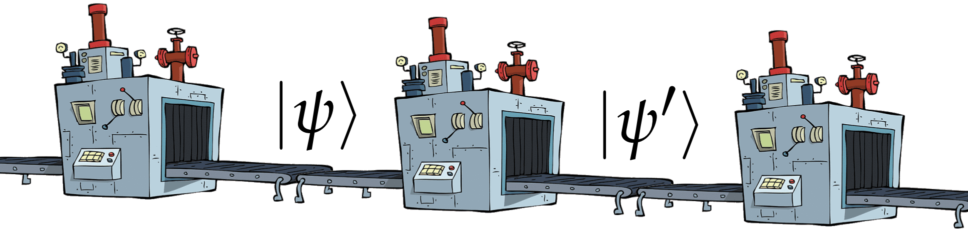

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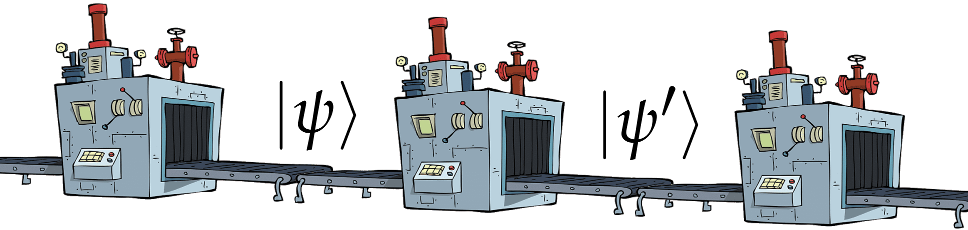
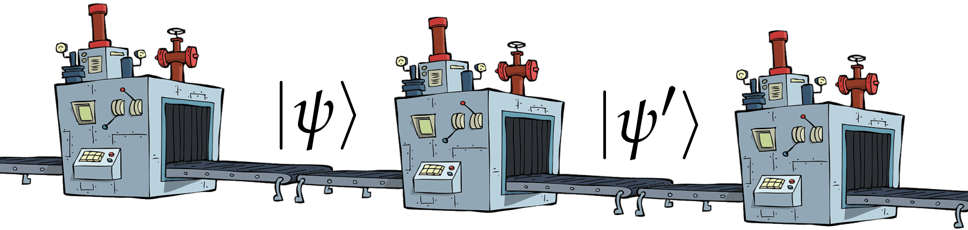


- quantum Fourier transform
- Grover iteration
- swap test
- …

Manifesto (lite)

New *quantum* primitives!



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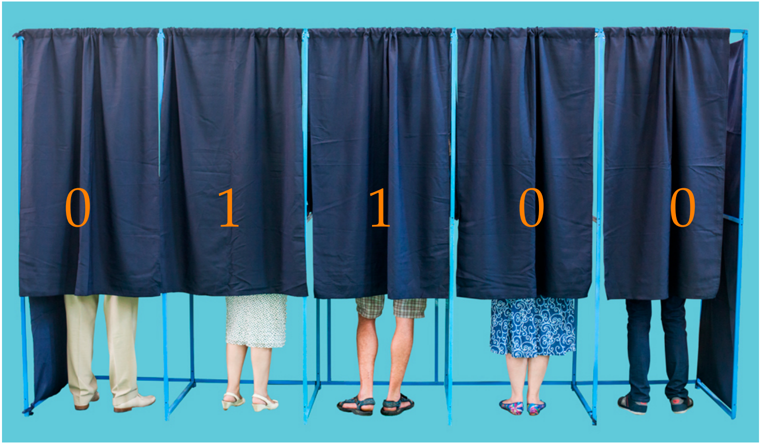
Majority vote



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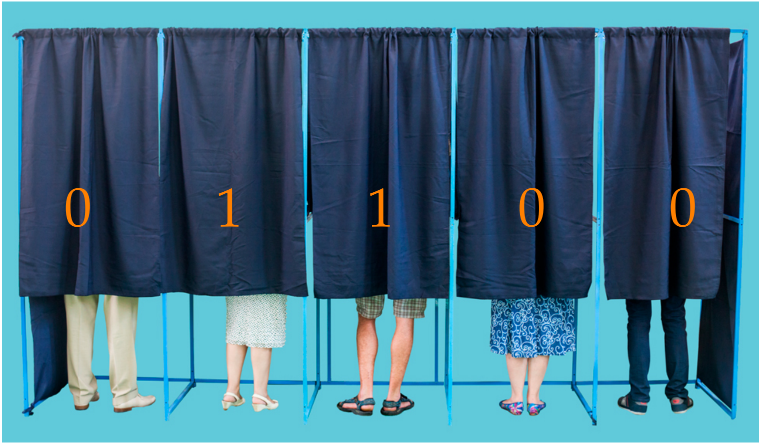
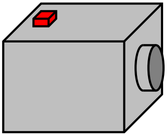


- success amplification
- error correction
- democracy

Quantum majority vote



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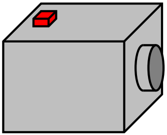
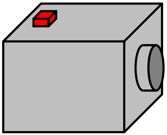


$|\psi\rangle$

Quantum majority vote



$|\psi\rangle|\psi\rangle$

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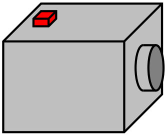
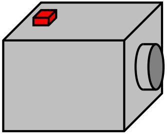


$|\psi\rangle|\psi\rangle|\psi^{\perp}\rangle$

Quantum majority vote



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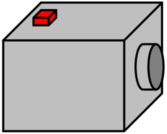

$$|\psi\rangle|\psi\rangle|\psi^{\perp}\rangle|\psi\rangle|\psi\rangle|\psi^{\perp}\rangle|\psi\rangle|\psi\rangle|\psi\rangle$$

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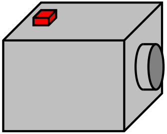

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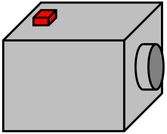

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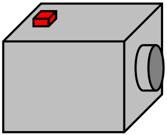
Computation in an unknown basis

Quantum majority vote



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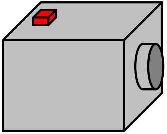

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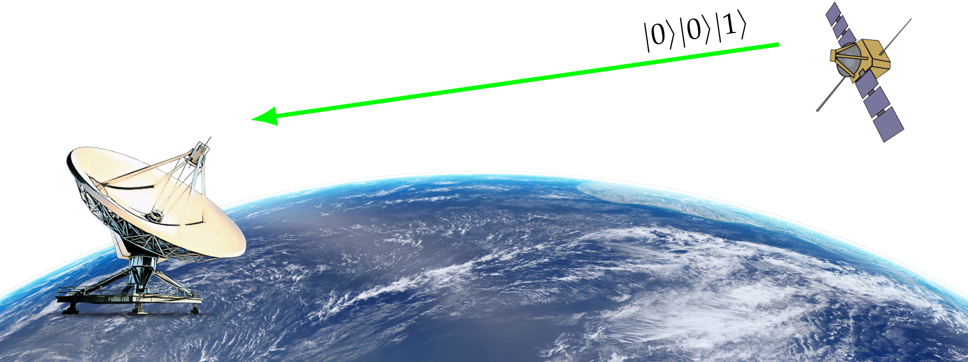

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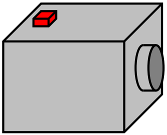

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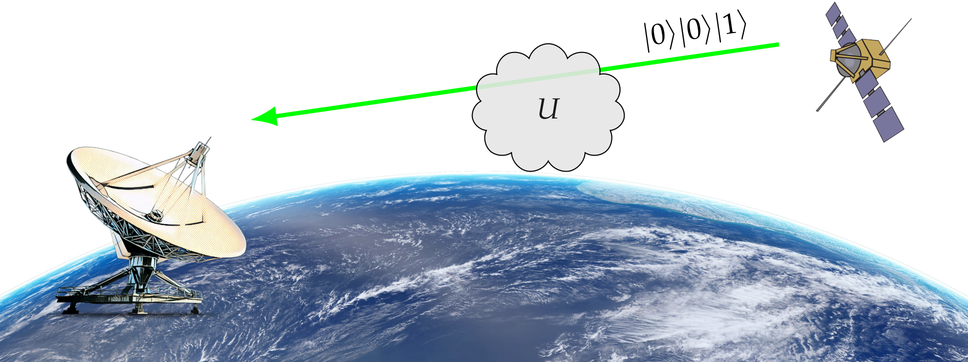

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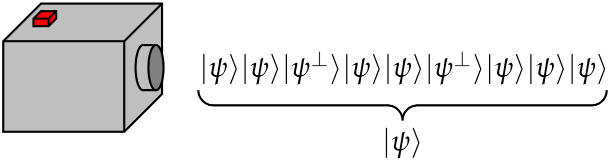

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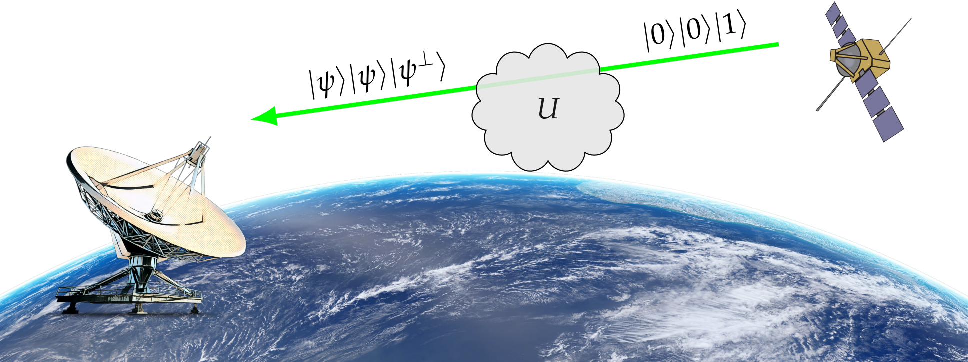

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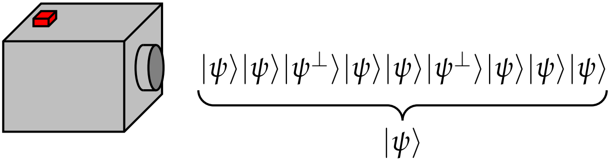

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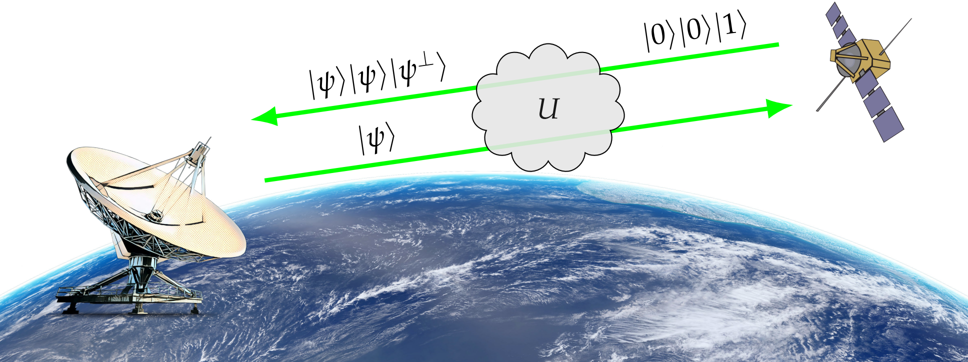

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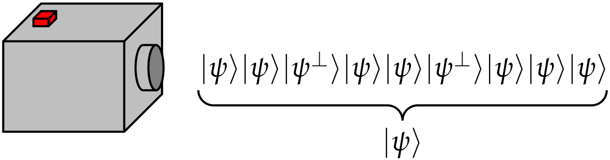

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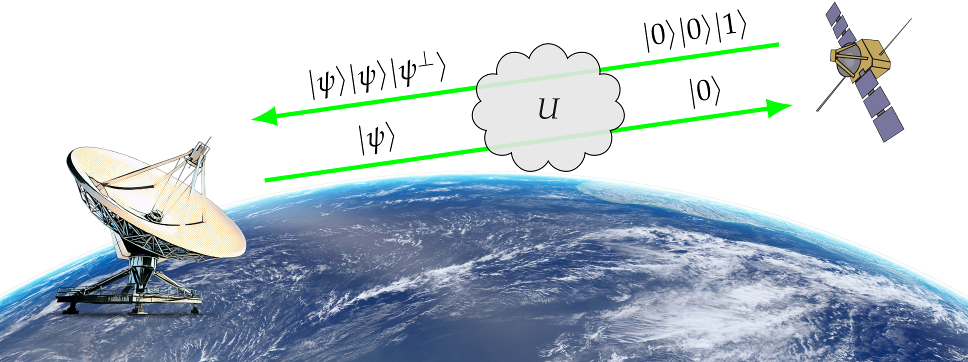

Equivariance

Unitary equivariance

Computing $f : \{0,1\}^n \to \{0,1\}$ in a *unitary-equivariant* way:

$$|x\rangle \mapsto \quad |f(x)\rangle \qquad \forall x \in \{0,1\}^n$$

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Windows

An error has occurred. To continue:

Press Enter to return to Windows, or

Press CTRL+ALT+DEL to restart your computer. If you do this,
you will lose any unsaved information in all open applications.

Error: 0E : 016F : BFF9B3D4

Press any key to continue _

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Equivariant functions

$f : \{0,1\}^n \to \{0,1\}$ is *equivariant* (or *self-dual*) if

$$f(\overline{x}) = \overline{f(x)}$$

Symmetric functions

Assumption

$f(x)$ depends only on the Hamming weight of $x \in \{0,1\}^n$

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Example: $n = 3$

$\lvert x \rvert$	0	1	2	3
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Even n is bad

$$f(0,1) = \overline{f(1,0)} = \overline{f(0,1)}$$

Problem

Assumptions

Given $f : \{0,1\}^n \to \{0,1\}$, assume

- f is equivariant
- f is symmetric
- n is odd

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Worst-case fidelity

$$F_f := \max_{\Phi} \min_{x,U} \langle f(x)|U^\dagger \Phi\Big(U^{\otimes n}|x\rangle\langle x|U^{\dagger\otimes n}\Big) U|f(x)\rangle$$

Results on majority

Trivial strategy: Output any qubit at random

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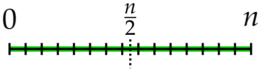
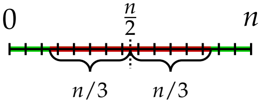
No promise



Promise



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	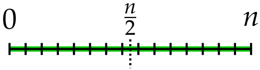	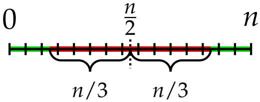
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	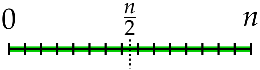	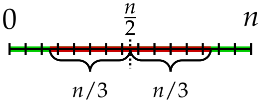
Trivial	$\frac{1}{2} + \frac{1}{2n}$	$\frac{5}{6}$
Optimal	$\frac{1}{2} + \Theta\left(\frac{1}{\sqrt{n}}\right)$	$1 - \Theta\left(\frac{1}{n}\right)$

Symmetric / anti-symmetric subspace of $\mathbb{C}^2 \otimes \mathbb{C}^2$

Symmetric states:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\otimes 2} |00\rangle = \begin{pmatrix} a^2 \\ ac \\ ca \\ c^2 \end{pmatrix} = a^2 \ |00\rangle \ + \sqrt{2}ac \ \frac{|01\rangle + |10\rangle}{\sqrt{2}} \ + c^2 \ |11\rangle$$

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Schur transform on two qubits

$$U_{\mathrm{Sch}} := \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$U_{\mathrm{Sch}} \, \mathrm{SWAP} \, U_{\mathrm{Sch}}^{\dagger} = \mathrm{diag}(-1, 1, 1, 1)$$

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$$U_{\text{Sch}} \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\otimes 2} U_{\text{Sch}}^{\dagger} = \begin{pmatrix} ad - bc & 0 & 0 & 0 \\ 0 & a^2 & \sqrt{2}ab & b^2 \\ 0 & \sqrt{2}ac & ad + bc & \sqrt{2}bd \\ 0 & c^2 & \sqrt{2}cd & d^2 \end{pmatrix}$$

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Each block defines a homomorphism: $Q(MN) = Q(M)Q(N)$

Schur transform on n qubits

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Can be implemented with $O(n^4 \log n)$ gates
(Kirby & Strauch, 2017)

Algorithm

$$U^{\otimes n}|x\rangle\langle x|U^{\dagger\otimes n}$$

Input: $U^{\otimes n}|x\rangle$ with unknown $x \in \{0,1\}^n$ and $U \in \mathrm{U}(2)$

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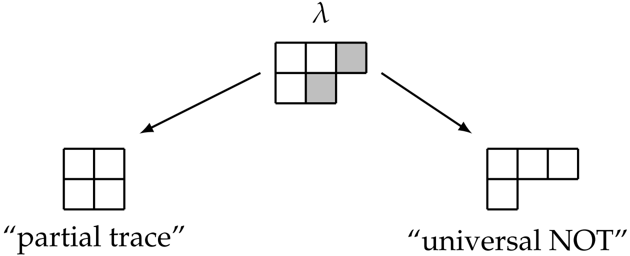
- Step 4 acts on a state of random dimension m_λ

Extremal equivariant channels

For each $\lambda \vdash n$, only two *extremal* unitary-equivariant $\mathbb{C}^{m_\lambda} \to \mathbb{C}^2$ channels:



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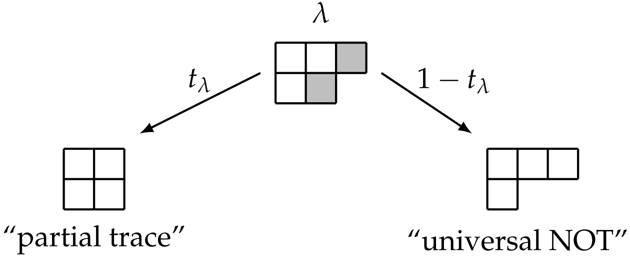

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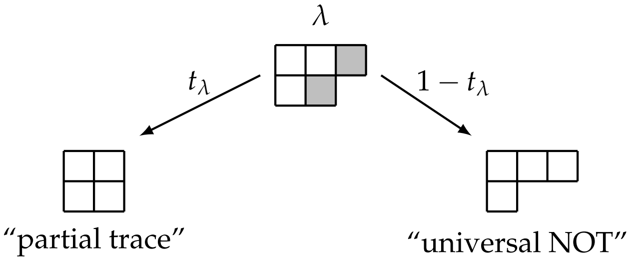

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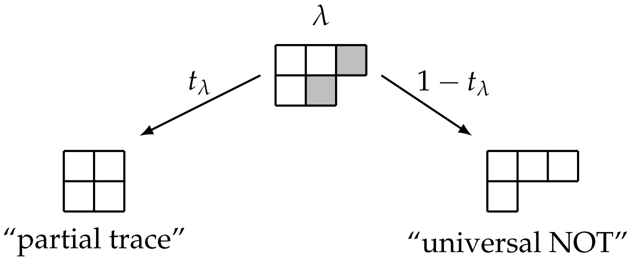

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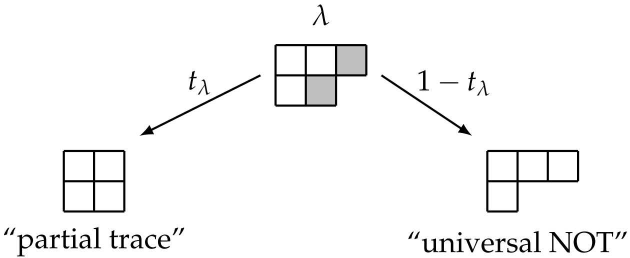

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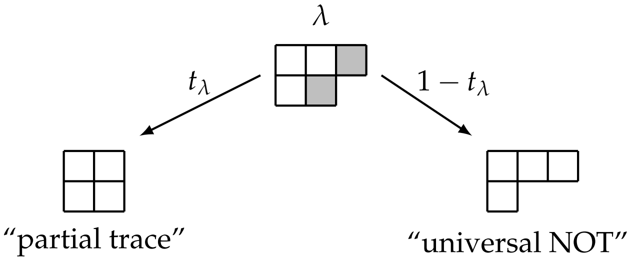

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Main result

Theorem (with Buhrman, Linden, Mančinska, Montanaro)

For any symmetric and equivariant n-bit boolean function f

- *the optimal parameters t_λ and the resulting fidelity can be determined by a linear program of size $n/2$*

Main result

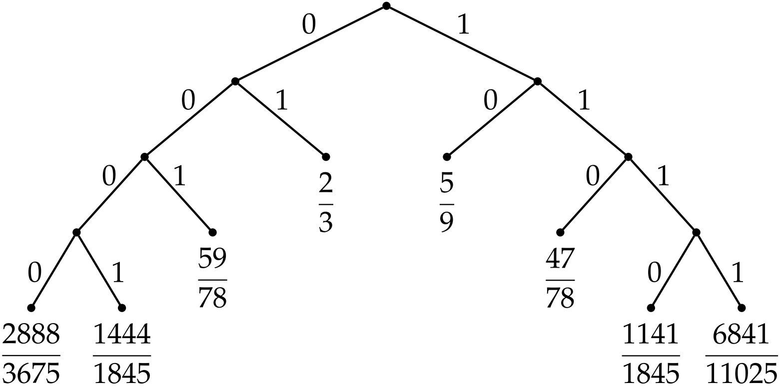
Theorem (with Buhrman, Linden, Mančinska, Montanaro)

For any symmetric and equivariant n-bit boolean function f

- *the optimal parameters t_λ and the resulting fidelity can be determined by a linear program of size $n/2$*
- *optimal quantum algorithm with $O(n^4 \log n)$ gates*

Fidelities of all 7-argument functions

$\lvert x \rvert$	0	1	2	3	4	5	6	7
$f(x)$	0	1	0	0	1	1	0	1



Fidelity depends only on the gap around $n/2$ in the truth table

Mathematical essence of the problem

Variables

- $c \in \mathbb{R}$
- $\Phi : \mathbb{C}^{2^n \times 2^n} \to \mathbb{C}^{2 \times 2}$

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Choi matrix

- Any linear map $\Phi : \mathrm{Mat}(\mathcal{H}_{\text{in}}) \to \mathrm{Mat}(\mathcal{H}_{\text{out}})$ can be represented by its *Choi matrix* $J(\Phi) \in \mathrm{Mat}(\mathcal{H}_{\text{in}} \otimes \mathcal{H}_{\text{out}})$:

$$J(\Phi) := \sum_{i,j=1}^{\dim \mathcal{H}_{\text{in}}} |i\rangle\langle j| \otimes \Phi\big(|i\rangle\langle j|\big)$$

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- Characterization of quantum channels:

$$J \succeq 0 \quad \text{(CP)} \qquad\qquad \mathrm{Tr}_{\mathcal{H}_{\mathrm{out}}} J = I_{\mathcal{H}_{\mathrm{in}}} \quad \text{(TP)}$$

Unitary equivariance

- Let Φ be a channel from n to m systems of dimension d:

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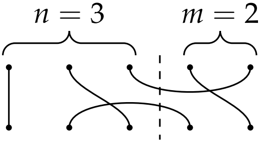
- What is the commutant of $U^{\otimes n} \otimes \bar{U}^{\otimes m}$?

Walled Brauer algebra

- $\mathcal{B}^{d}_{n,m}$ consists of formal linear combinations of diagrams:



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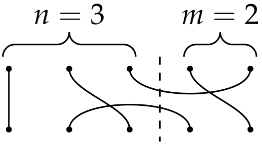

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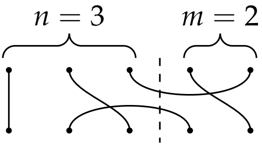

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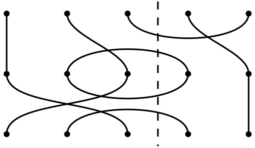

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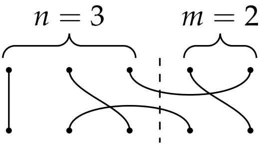

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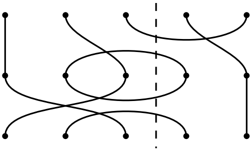
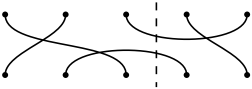


$\sigma_1\,\sigma_2 = d \cdot$



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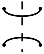
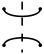
- Contraction:

$\psi\Big($  $\Big) : |i\rangle|j\rangle \mapsto \delta_{i,j} \sum_{k=1}^{d} |k\rangle|k\rangle,$ $\quad \psi\Big($  $\Big) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$

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Theorem (Koike 1989, Benkart et al. 1994)

The commutant of $U^{\otimes n} \otimes \bar{U}^{\otimes m}$ *is equal to* $\psi(\mathcal{B}^d_{n,m})$

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The following are equivalent:

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Key insight

When additional permutational symmetry is imposed, each block $J_\lambda(\Phi)$ becomes diagonal

Semidefinite programming with diagrams

Unitary-equivariant SDP

$$\begin{aligned}
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\text{s.t.} \quad & \mathrm{Tr}(A_k X) \leq b_k, & \forall k \in [k_1] \\
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Theorem (with Grinko)

Assuming additional permutational symmetry, the above SDP can be converted to an equivalent LP with at most $l \leq N$ variables and $k_1 + k_2 N + l$ constraints where $N := (n+m)!$

Connection to Baltic countries

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- Jucys–Murphy elements of $\mathbb{C}S_n$ are central in the Vershik–Okounkov approach to representation theory of S_n

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Thank you!